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What is claimed is:

1. An apparatus for computing multiple integral of a multidimensional integrand function A to be integrated with using a vector map f with unbounded support which converts an m ($m \geq 1$) -dimensional vector having real number components into an m-dimensional vector having real number components, by which a multidimensional density function ρ for the limiting distribution resulting from repeatedly applying the map f to an m-dimensional vector u is analytically solvable, said apparatus comprising:

a first storage unit which stores an m-dimensional vector $x = (x_1, x_2, \ldots, x_m)$;

a second storage unit which stores a scalar value w;

a first computing unit which computes a vector $x' = f(x) = (x'_1, x'_2, \ldots, x'_m)$ resulting from applying said vector map f to said vector x being stored in said first storage unit;

a second computing unit which computes a scalar value $w' = A(x)/\rho(x)$ based on said vector x being stored in said first storage unit and said scalar value w being stored in said second storage unit;

an update unit which updates the value stored in said first storage unit by storing said vector x' computed by said first computing unit on said first storage unit, and updates the value in said second storage unit by adding said scalar value w' computed by said second computing unit to a value to be stored in said second storage unit; and

an output unit which computes a scalar value $s = w/(c+1)$ based on said scalar value w being stored in said second storage unit when the number of update times by said update unit becomes c ($c \geq 1$), and outputs said scalar value s as a result of the multiple integral.

2. The apparatus according to claim 1, wherein said scalar value stored in said second storage unit first is a result from dividing a value resulting from applying said function A to said m-dimensional vector stored in said first storage unit first by a value resulting from applying said density function ρ to said m-dimensional vector stored in said first storage unit first.

3. The apparatus according to claim 1, wherein said scalar value stored in said second storage unit first is 0, and

said output unit computes a scalar value $s' = w/c$, and outputs said scalar value s' as the result of the multiple integral instead of said scalar value s.

4. The apparatus according to claim 1 further comprising:

a convergence rate obtainer which obtains convergence rate of scalar values sequentially output by said output unit while varying said number of update times c for each of plural vector maps $g_1, g_2, \ldots, g_k$ ($k \geq 2$) which are prepared as said vector map f;

a vector map selector which refers to the convergence rates obtained by said convergence rate obtainer, and selects a vector map g_h ($1 \leq h \leq k$) which shows fastest convergence rate; and

an output controller which controls said output unit to output said scalar value with using said vector map g_h as said vector map f and the number of update times c' ($c' > c$) instead of said number of update times c.

5. The apparatus according to claim 1, wherein a multidimensional density function ρ representing the limiting distribution of a vector sequence

$$u,\ f(u),\ f(f(u)),\ f(f(f(u))),\ \ldots$$

resulting from applying said vector map f to a predetermined m-dimensional vector $u = (u_1, u_2, \ldots, u_m)$ for equal to or greater than 0 times, satisfies the following property of:

$$\rho(u) = \textstyle\prod_{i=0}^{m} \rho_i(u_i);$$

$$\rho_i(u_i) \sim c_{-i}\,|u_i|^{-(1+a)} \quad \text{for } u_i \rightarrow -\infty;$$

$$\rho_i(u_i) \sim c_{-i}\,|u_i|^{-(1+a)} \quad \text{for } u_i \rightarrow +\infty;$$

$$(a>0,\ 1 \leq i \leq m,\ c_{-i}>0,\ c_{+i}>0)$$

6. The apparatus according to claim 5, wherein said vector map f is defined as

$$f(u) = (f_1(u_1),\ f_2(u_s),\ \ldots,\ f_m(u_m))$$

by a function $f_i(t) = g_i(d_i\, t)/d_i$ ($d_i > 0$) which is defined in $1 \leq i \leq m$, and said map g_i is defined by any one of the following maps φ_j ($1 \leq j \leq 8$) and a natural number n_i ($n_i \geq 2$), as

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follows:

$g_i(\varphi_j(\theta)) = \varphi_j(n_i\theta);$

$\varphi_1(\theta) = -\mathrm{sgn}(\tan\theta) / |\tan\theta|^{1/a};$

$\varphi_2(\theta) = -\mathrm{sgn}(\tan\theta) \times |\tan\theta|^{1/a};$

$\varphi_3(\theta) = -\mathrm{sgn}(\cos\theta) / |\tan\theta|^{1/a};$

$\varphi_4(\theta) = -\mathrm{sgn}(\cos\theta) \times |\tan\theta|^{1/a};$

$\varphi_5(\theta) = \mathrm{sgn}(\cos\theta) / |\tan\theta|^{1/a};$

$\varphi_6(\theta) = \mathrm{sgn}(\cos\theta) \times |\tan\theta|^{1/a};$

$\varphi_7(\theta) = \mathrm{sgn}(\sin\theta) / |\tan\theta|^{1/a};$

$\varphi_8(\theta) = \mathrm{sgn}(\sin\theta) \times |\tan\theta|^{1/a};$

$\mathrm{sgn}(t) = 1 \quad \text{for } t > 0;$

$\mathrm{sgn}(t) = 0 \quad \text{for } t = 0;$

$\mathrm{sgn}(t) = -1 \quad \text{for } t < 0$

7. The apparatus according to claim 5 further comprising:

a convergence rate obtainer which defines said map f for each of plural positive numbers $q_1, q_2, \ldots, q_k$ ($k \geq 2$) prepared as an invariable a, and obtains convergence rates of the scalar values sequentially output by said output unit while varying said number of update times c;

a positive number selector which refers the convergence rates obtained by said convergence rate obtainer, and selects an integer q_h ($1 \leq h \leq k$) which shows the fastest convergence rate; and

an output controller which defines said map f with using said positive number q_h as said invariable a, and controls said output unit to output said scalar values with using the number of update times c' (c'>c) instead of said number of update times c.

8. The apparatus according to claim 6 further comprising:

a convergence rate obtainer which defines said map g_i with using plural ones of said maps φ_j, and obtains convergence rates of the scalar values sequentially output by said

output unit while varying said number of update times c;

a map selector which refers to the convergence rates obtained by said convergence rate obtainer, and selects one of said maps φ_j which shows the fastest convergence rate; and

an output controller which defines said map g_i with using said map φ_j selected by said map selector, and controls said output unit to output the scalar values with using the number of update times c' (c'>c) instead of said number of update times c.

9. The apparatus according to claim 6 further comprising:

a convergence rate obtainer which defines said map g_i relating to each of plural natural numbers $p_1, p_2, \ldots, p_k$ ($k \geq 2$) as said natural numbers n_i, and obtains convergence rates of the scalar values sequentially output by said output unit while varying said number of update times c;

a natural number selector which refers to the convergence rates obtained by said convergence rate obtainer, and selects a natural number p_h ($1 \leq h \leq k$) which shows the fastest convergence rate; and

an output controller which defines said natural number map g_i with using said natural number p_h as said natural number n_i, and controls said output unit to output the scalar values with using the number of update times c' (c'>c) instead of said number of update times c.

10. The apparatus according to claim 1, wherein said output unit computes said scalar value s each time said update unit carries out update, compares said latest scalar value s with the former scalar value which is computed at former update, and outputs said latest scalar value s if a result of the comparison satisfies a predetermined condition for terminating the computation.

11. A method for computing multiple integral of a multidimensional integrand function A to be integrated with using a vector map f with unbounded support which converts an m ($m \geq 1$) -dimensional vector having real number components into an m-

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dimensional vector having real number components, by which a multidimensional density function ρ for the limiting distribution resulting from repeatedly applying the map f to an m-dimensional vector u is analytically solvable,

a first storage unit which stores an m-dimensional vector $x = (x_1, x_2, \ldots, x_m)$, and

a second storage unit which stores a scalar value w,

said method comprising the steps of:

computing a vector $x' = f(x) = (x'_1, x'_2, \ldots, x'_m)$ resulting from applying said vector map f to said vector x being stored in said first storage unit;

computing a scalar value $w' = A(x)/\rho(x)$ based on said vector x being stored in said first storage unit and said scalar value w being stored in said second storage unit;

updating the value stored in said first storage unit by storing said vector x' computed by said first computing unit on said first storage unit, and updating the value in said second storage unit by adding said scalar value w' computed by said second computing unit to a value to be stored on said second storage unit; and

computing a scalar value $s = w/(c+1)$ based on said scalar value w being stored in said second storage unit when the number of update times by said update unit becomes c ($c \geq 1$), and outputting said scalar value s as a result of the multiple integral.

12. The method according to claim 11, wherein said scalar value stored in said second storage unit first is a result from dividing a value resulting from applying said function A to said m-dimensional vector stored in said first storage unit first by a value resulting from applying said density function ρ to said m-dimensional vector stored in said first storage unit first.

13. The method according to claim 11, wherein said scalar value stored in said second storage unit first is 0, and

said outputting step computes a scalar value $s' = w/c$ and outputs said scalar value s' as the result of the multiple integral instead of said scalar value s.

14. The method according to claim 11 further comprising the steps of:

obtaining convergence rate of scalar values sequentially output by said outputting step while varying said number of update times c for each of plural vector maps g_1, $g_2, \ldots, g_k$ ($k \geq 2$) which are prepared as said vector map f; and

referring to the convergence rates obtained by said convergence rate obtainer, and selecting a vector map g_h ($1 \leq h \leq k$) which shows fastest convergence rate, and

said outputting step outputs said scalar value with using said vector map g_h as said vector map f and the number of update times c' (c'>c) instead of said number of update times c.

15. The method according to claim 11, wherein a multidimensional density function ρ representing the limiting distribution of a vector sequence

$$u, f(u), f(f(u)), f(f(f(u))), \ldots$$

resulting from applying said vector map f to a predetermined m-dimensional vector $u = (u_1, u_2, \ldots, u_m)$ for equal to or greater than 0 times, satisfies the following property of:

$$\rho(u) = \prod_{i=0}^{m} \rho_i(u_i);$$

$$\rho_i(u_i) \sim c_{-i} |u_i|^{-(1+a)} \quad \text{for } u_i \to -\infty;$$

$$\rho_i(u_i) \sim c_{-i} |u_i|^{-(1+a)} \quad \text{for } u_i \to +\infty;$$

$$(a>0,\ 1 \leq i \leq m,\ c_{-i}>0,\ c_{+i}>0)$$

16. The method according to claim 15, wherein said vector map f is defined as

$$f(u) = (f_1(u_1), f_2(u_s), \ldots, f_m(u_m))$$

by a function $f_i(t) = g_i(d_i\, t)/d_i$ ($d_i>0$) which is defined in $1 \leq i \leq m$, and said map g_i is defined by any one of the following maps φ_j ($1 \leq j \leq 8$) and a natural number n_i ($n_i \geq 2$), as follows:

$g_i(\varphi_j(\theta)) = \varphi_j(n_i\theta)$;

$\varphi_1(\theta) = -\mathrm{sgn}(\tan\theta) / |\tan\theta|^{1/a}$;

$\varphi_2(\theta) = -\mathrm{sgn}(\tan\theta) \times |\tan\theta|^{1/a}$;

$\varphi_3(\theta) = -\mathrm{sgn}(\cos\theta) / |\tan\theta|^{1/a}$;

$\varphi_4(\theta) = -\mathrm{sgn}(\cos\theta) \times |\tan\theta|^{1/a}$;

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$\varphi_5(\theta) = \mathrm{sgn}(\cos\theta) / |\tan\theta|^{1/a}$;

$\varphi_6(\theta) = \mathrm{sgn}(\cos\theta) \times |\tan\theta|^{1/a}$;

$\varphi_7(\theta) = \mathrm{sgn}(\sin\theta) / |\tan\theta|^{1/a}$;

$\varphi_8(\theta) = \mathrm{sgn}(\sin\theta) \times |\tan\theta|^{1/a}$;

$\mathrm{sgn}(t) = 1$ for $t > 0$;

$\mathrm{sgn}(t) = 0$ for $t = 0$;

$\mathrm{sgn}(t) = -1$ for $t < 0$

17. The method according to claim 15 further comprising the step of:

defining said map f for each of plural positive numbers $q_1, q_2, \ldots, q_k$ ($k \geq 2$) prepared as an invariable a, and obtaining convergence rates of the scalar values sequentially output by said output unit while varying said number of update times c;

referring to the obtained convergence rates, and selecting a positive number q_h ($1 \leq h \leq k$) which shows the fastest convergence rate; and

defining said map f with using said positive number q_h as said invariable a, and controlling output of said scalar values with using the number of update times c' (c'>c) instead of said number of update times c.

18. The method according to claim 16 further comprising the steps of:

defining said map g_i with using plural ones of said maps φ_j, and obtaining convergence rates of the scalar values sequentially output by said output step while varying said number of update times c;

referring to the obtained convergence rates, and selecting one of said maps φ_j which shows the fastest convergence rate; and

defining said map g_i with using said selected map φ_j selected, and controlling output of the scalar values with using the number of update times c' (c'>c) instead of said number of update times c.

19. The method according to claim 16 further comprising the steps of:

defining said map g_i relating to each of plural natural numbers $p_1, p_2, \ldots, p_k$ ($k \geq 2$)

as said natural numbers n_i, and obtaining convergence rates of the scalar values sequentially output by said outputting step while varying said number of update times c;

referring to the obtained convergence rates, and selecting a natural number p_h ($1 \le h \le k$) which shows the fastest convergence rate; and

defining said natural number map g_i with using said natural number p_h as said natural number n_i, and controlling output of the scalar values with using the number of update times c' (c'>c) instead of said number of update times c.

20. The method according to claim 11, wherein said outputting step computes said scalar value s each time said updating step carries out update, compares said latest scalar value s with the former scalar value which is computed at former update, and outputs said latest scalar value s if a result of the comparison satisfies a predetermined condition for terminating the computation.

21. A computer readable recording medium storing a program for computing multiple integral of a multidimensional integrand function A to be integrated with using a vector map f with unbounded support which converts an m ($m \ge 1$) -dimensional vector having real number components into an m-dimensional vector having real number components, by which a multidimensional density function ρ for the limiting distribution resulting from repeatedly applying the map f to an m-dimensional vector u is analytically solvable, said program causes a computer to function as:

a first storage unit which stores an m-dimensional vector $x = (x_1, x_2, \ldots, x_m)$;

a second storage unit which stores a scalar value w;

a first computing unit which computes a vector $x' = f(x) = (x'_1, x'_2, \ldots, x'_m)$ resulting from applying said vector map f to said vector x being stored in said first storage unit;

a second computing unit which computes a scalar value $w' = A(x)/\rho(x)$ based on said vector x being stored in said first storage unit and said scalar value w being stored in said second storage unit;

an update unit which updates the value stored in said first storage unit by storing

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said vector x' computed by said first computing unit on said first storage unit, and updates the value in said second storage unit by adding said scalar value w' computed by said second computing unit to a value to be stored on said second storage unit; and

an output unit which computes a scalar value $s = w/(c+1)$ based on said scalar value w being stored in said second storage unit when the number of update times by said update unit becomes c ($c \geq 1$), and outputs said scalar value s as a result of the multiple integral.

22. The recording medium according to claim 21, wherein said scalar value stored in said second storage unit first is a result from dividing a value resulting from applying said function A to said m-dimensional vector stored in said first storage unit first by a value resulting from applying said density function ρ to said m-dimensional vector stored in said first storage unit first.

23. The recording medium according to claim 21, wherein said scalar value stored in said second storage unit first is 0, and

said output unit computes a scalar value $s' = w/c$, and outputs said scalar value s' as the result of the multiple integral instead of said scalar value s.

24. The recording medium according to claim 21, wherein said program further causes said computer to function as:

a convergence rate obtainer which obtains convergence rate of scalar values sequentially output by said output unit while varying said number of update times c for each of plural vector maps $g_1, g_2, \ldots, g_k$ ($k \geq 2$) which are prepared as said vector map f;

a vector map selector which refers to the convergence rates obtained by said convergence rate obtainer, and selects a vector map g_h ($1 \leq h \leq k$) which shows fastest convergence rate; and

an output controller which controls said output unit to output said scalar value with using said vector map g_h as said vector map f and the number of update times c' (c'>c) instead of said number of update times c.

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25. The recording medium according to claim 21, wherein a multidimensional density function ρ representing the limiting distribution of a vector sequence

$$u,\ f(u),\ f(f(u)),\ f(f(f(u))),\ \ldots$$

resulting from applying said vector map f to a predetermined m-dimensional vector $u = (u_1, u_2, \ldots, u_m)$ for equal to or greater than 0 times, satisfies the following property of:

$$\rho(u) = \prod_{i=0}^{m} \rho_i(u_i);$$

$$\rho_i(u_i) \sim c_{-i}\, |u_i|^{-(1+a)} \quad \text{for } u_i \to -\infty;$$

$$\rho_i(u_i) \sim c_{-i}\, |u_i|^{-(1+a)} \quad \text{for } u_i \to +\infty;$$

$$(a>0,\ 1\leq i\leq m,\ c_{-i}>0,\ c_{+i}>0)$$

26. The recording medium according to claim 25, wherein said vector map f is defined as

$$f(u) = (f_1(u_1),\ f_2(u_s),\ \ldots,\ f_m(u_m))$$

by a function $f_i(t) = g_i(d_i\, t)/d_i$ $(d_i>0)$ which is defined in $1\leq i\leq m$, and said map g_i is defined by any one of the following maps φ_j $(1\leq j\leq 8)$ and a natural number n_i $(n_i\geq 2)$, as follows:

$$g_i(\varphi_j(\theta)) = \varphi_j(n_i\theta);$$

$$\varphi_1(\theta) = -\mathrm{sgn}(\tan\theta) / |\tan\theta|^{1/a};$$

$$\varphi_2(\theta) = -\mathrm{sgn}(\tan\theta) \times |\tan\theta|^{1/a};$$

$$\varphi_3(\theta) = -\mathrm{sgn}(\cos\theta) / |\tan\theta|^{1/a};$$

$$\varphi_4(\theta) = -\mathrm{sgn}(\cos\theta) \times |\tan\theta|^{1/a};$$

$$\varphi_5(\theta) = \mathrm{sgn}(\cos\theta) / |\tan\theta|^{1/a};$$

$$\varphi_6(\theta) = \mathrm{sgn}(\cos\theta) \times |\tan\theta|^{1/a};$$

$$\varphi_7(\theta) = \mathrm{sgn}(\sin\theta) / |\tan\theta|^{1/a};$$

$$\varphi_8(\theta) = \mathrm{sgn}(\sin\theta) \times |\tan\theta|^{1/a};$$

$$\mathrm{sgn}(t) = 1 \quad \text{for } t > 0;$$

$$\mathrm{sgn}(t) = 0 \quad \text{for } t = 0;$$

$$\mathrm{sgn}(t) = -1 \quad \text{for } t < 0$$

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27. The recording medium according to claim 25, wherein said program further causes said computer to function as:

a convergence rate obtainer which defines said map f for each of plural positive numbers $q_1, q_2, \ldots, q_k$ ($k \geq 2$) prepared as an invariable a, and obtains convergence rates of the scalar values sequentially output by said output unit while varying said number of update times c;

an integer selector which refers the convergence rates obtained by said convergence rate obtainer, and selects a positive number q_h ($1 \leq h \leq k$) which shows the fastest convergence rate; and

an output controller which defines said map f with using said positive number q_h as said invariable a, and controls said output unit to output said scalar values with using the number of update times c' (c'>c) instead of said number of update times c.

28. The recording medium according to claim 26, wherein said program further causes said computer to function as:

a convergence rate obtainer which defines said map g_i with using plural ones of said maps φ_j, and obtains convergence rates of the scalar values sequentially output by said output unit while varying said number of update times c;

a map selector which refers to the convergence rates obtained by said convergence rate obtainer, and selects one of said maps φ_j which shows the fastest convergence rate; and

an output controller which defines said map g_i with using said map φ_j selected by said map selector, and controls said output unit to output the scalar values with using the number of update times c' (c'>c) instead of said number of update times c.

29. The recording medium according to claim 26, wherein said program further causes said computer to function as:

a convergence rate obtainer which defines said map g_i relating to each of plural natural numbers $p_1, p_2, \ldots, p_k$ ($k \geq 2$) as said natural numbers n_i, and obtains convergence

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rates of the scalar values sequentially output by said output unit while varying said number of update times c;

a natural number selector which refers to the convergence rates obtained by said convergence rate obtainer, and selects a natural number p_h ($1 \leq h \leq k$) which shows the fastest convergence rate; and

an output controller which defines said natural number map g_i with using said natural number p_h as said natural number n_i, and controls said output unit to output the scalar values with using the number of update times c' (c'>c) instead of said number of update times c.

30. The recording medium according to claim 21, wherein said output unit computes said scalar value s each time said update unit carries out update, compares said latest scalar value s with the former scalar value which is computed at former update, and outputs said latest scalar value s if a result of the comparison satisfies a predetermined condition for terminating the computation.

31. The recording medium according to claim 21, wherein said recording medium is a compact disc, a floppy disk, a hard disk, a magneto-optical disk, a digital versatile disc, a magnetic tape, or a semiconductor memory.